STUDIA UBB PSYCHOL.-PAED., LVII, 1, 2012, pp. 97 – 107

HOW PRIMARY SCHOOL TEACHERS DEVELOP THEIR PUPILS' MATHEMATICAL WORD PROBLEM SOLVING SKILLS

IULIANA MARCHIŞ*

ABSTRACT. Problem solving is an important skill that all persons should possess. Mathematical word problems contribute for the development of the problem solving skill. These problems occupy an important space in primary school mathematical curriculum. Primary school teachers have to develop their pupils' mathematical word problem solving skills. The aim of this research is to study how primary school teachers solve mathematical word problems with their pupils. Three quarters of the teachers guide pupils in order to understand the problem and encourage them for self-control during problem solving. Only one third of the respondents encourage their students to solve the problems with more methods. Only half of the teachers ask their pupils to present the solution. Three quarters of the respondents have a positive attitude and guidance in case their pupils can't solve a problem. Almost three quarters of the primary school teachers give interesting, real life close problems in class. Teachers expect more from their pupils as regarding problem solving behaviour than they do.

Keywords: *teaching Mathematics, primary school, problem solving, mathematical word problem.*

Introduction

Mathematical word problems occupy an important space in primary school mathematical curriculum. Thus is because these problems essentially contribute to the development of problem solving skills.

Mathematics education has undergone major changing in the last decades. The focus has moved from acquiring mathematical knowledge to developing problem solving skills. This change is reflected also in international tests (PISA, TIMSS, etc.) where emphasis is put on solving mathematical problems arisen from real world situations. Teachers who can't adapt to these changes probably will produce students who can only use the learnt rules, formulas, or methods (Ernest, 1988). There are many reasons that teachers can't make this change. They may not master the pedagogical skills or/and confidence to adapt to these changes (Gregg, 1995) or they may have a not too strong mathematical background (Brown, Cooney & Jones,

* Lecturer PhD, Pedagogy and Applied Didactics Department, Babes-Bolyai University, Cluj-Napoca, Romania, iuliana.marchis@ubbcluj.ro

1990). Even if they have the necessary pedagogical and mathematical knowledge, they are tempted to use teaching methods similar to those of their own teachers (Brown, Cooney & Jones, 1990). Also, teachers' beliefs about mathematics influence their teaching style. "One's conception of what mathematics is affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it. (Hersh, 1986, p. 13) Two teachers with similar mathematical and pedagogical knowledge could teach differently depending what they think important: the mastery of notions, formulas, methods and procedures or "the continually expanding field of human creation and invention" (Ernest, 1988, p. 93).

A previous paper highlighted that in Romania primary school teachers' mathematical problem solving skills and behaviour should be improved (Marchis, 2011).The aim of this article is to study how primary school teachers solve mathematical word problems with their pupils.

1. Theoretical background

1.1. Attitude to mathematical problem solving

Students' interest in mathematics, their beliefs in the utility of the mathematical knowledge in their future career or in their everyday life determine in a fundamental way their problem-solving behaviour. „Belief systems are one's mathematical world view, the perspective with which one approaches mathematics and mathematical task. One's beliefs about mathematics can determine how one chooses to approach a problem, which techniques will be used or avoided, how long and how hard one will work on it, and so on." (Schoenfeld, 1985, p. 45) There is a link between students' attitudes and their achievement in mathematics (Schoenfeld, 1989; Mc Leod, 1992; Brown et al. 1988), thus developing a positive attitude towards learning mathematics is important. First of all teachers' attitude towards mathematics and mathematics teaching has an important influence on pupils. Also, giving students interesting problems to solve, increase their motivation for learning mathematics. Teaching pupils how to solve mathematical problems develops a positive attitude towards word problem solving (Higgins, 1997). Composing their own word problems also helps students in changing their attitudes regarding these problems and becoming familiar with the mathematical terminology (Edwards et al., 2002).

1.2. Mathematical word problem solving

Mathematical word problem solving requires multiple processes, such as reading, text comprehension, problem representation, selection and execution of calculation operations (Kintsch & Greeno, 1985; Mayer & Hegarty, 1996; Swanson, 2004). Pólya (1945) has identified four main stages when solving a problem: understanding the problem, making a plan, carrying out the plan, and reviewing the solution. Similar steps are described by other researchers (among others Higgins, 1997; Leader & Middleton, 2004; Ridlon, 2004). According to Mayer (1983), problem solving has two phases: problem representation and search for solutions.

1.3. Understanding the text of the problem

When solving mathematical word problems, it is important to understand the text of the problem (DeCorte & Verschaffel, 1985; Kintsch & Greeno, 1985; Reusser, 1989; Vilenius-Tuohimaa, Acenola & Nurmi, 2008). The understanding stage includes some text comprehension techniques, for example, to identify the unknown words, to reformulate the problem, to think about a picture or diagram that might help to understand the problem context, and the relations between the given and unknown data (Pólya, 1957).

When reading a word problem, the difficulties encountered by the pupils could be related with understanding of some words used in the text, and understanding some sequences or some specific vocabulary (Ballew & Cunningham, 1982; Bernardo, 1999; Stape, 2011). If pupils are familiar with the story content, they understand the text better (Wiest, 1996). Visual representation has an important role in the organization of information given in the text (Antonietti, 1991; Hegarty, Mayer & Monk, 1995). Pupils should be taught how to solve a word problem: to read and understand the problem, to design a solution plan, to solve the problem, and than to formulate and check their answer in the context of the problem (Higgins, 1997; Ridlon, 2004).

2. Research

2.1. Research design

Aim of the research

The aim of the research is to study how primary school teachers develop their pupils' mathematical word problem solving skills.

Tool of the research

The main tool of the research is a questionnaire developed for evaluating how primary school teachers develop their pupils' problem solving skills. The first 3 items are demographic questions, the next 27 items are related with the topic of the research and they are affirmations which have to be evaluated by the respondents on a 5-point Likert scale: from 1- not at all typical for me to 5 – totally describes me. These items can be divided in two clusters: items related with teachers' problem solving behaviour (5 items) and questions related with teachers' actions in order to facilitate pupils' problem solving (22 items). There are only 5 questions related with teachers' problem solving, because the aim is not to study their problem solving behaviour. These items are only for comparing teachers' behaviour with their expectations in pupils' problem solving behaviour. The affirmations are formulated based on the theory related with problem solving. Cronbach's alpha reliability for the questionnaire is .0.823.

Sample of the research

The questionnaire was anonymously filled in by 31 primary school teachers during January-February 2011. All of the respondents are females. This reflects the reality in the system, there are only few male primary school teachers.

More than one third of the respondents (36%) have between 31 and 40 years old, 23% between 41 and 50 years old, 19-19 % less than 25 years old respectively between 25 and 30 years old (see Figure 1).

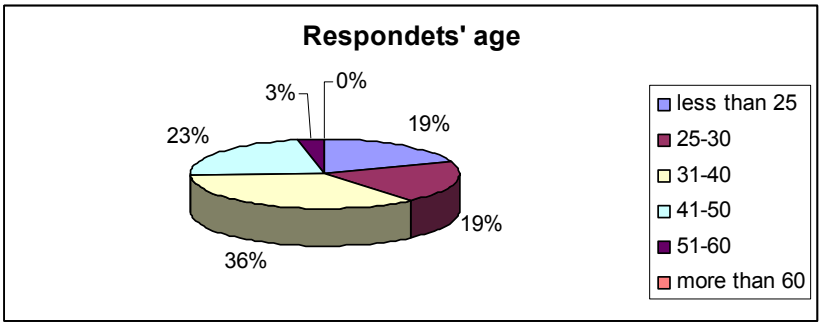

Figure 1. Respondents' age

Almost one third of the teachers (28%) have between 11 and 15 years of teaching experience, 26% between 2 and 6 years, 16% between 16 and 25 years of experience (for more details see Figure 2).

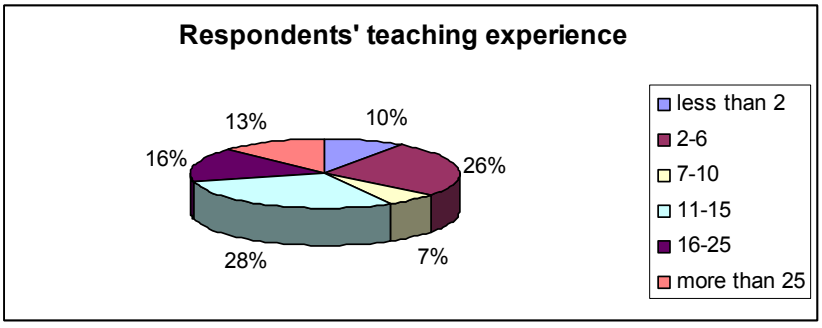

Figure 2. Respondents' teaching experience

2.2. Results

Items related with teachers' problem solving guidance are grouped in five clusters: understanding the text of the problem (Table 1), behaviour during problem solving (Table 2), presenting the solution (Table 3), actions in case of unsuccessful problem solving (Table 4), and criteria of selecting the problems given in the classroom (Table 5). Means, standard deviations, and percentages of those for who the affirmation is true are presented in Table 1, Table 2, Table 3, Table 4, and Table 5.

Table 1.

Understanding the text of the problem

Item	Mean	Standard deviation	Affirmation is true (%)
I ask pupils to read the text carefully.	4.61	.803	87.1
I ask pupils to reformulate the text of the problem with their own words.	3.97	1.016	74.2
I ask pupils to write the data and the relations between these data.	4.32	.945	74.2

Table 2.

Behaviour during problem solving

Item	Mean	Standard deviation	Affirmation is true (%)
I ask pupils to solve the problem with more methods.	2.94	1.031	38.7
I ask pupils to check if they have used all the data of the problem.	4.16	.969	67.7
I ask pupils to check if the solution is correct.	4.42	.765	83.9
I ask pupils to select the most efficient method, if the problem can be solved with more methods.	3.68	1.013	48.4

Table 3.

Presenting the solution

Item	Mean	Standard deviation	Affirmation is true (%)
I ask pupils to write down the detailed solution.	3.81	.873	58.1
I ask pupils to explain the solution in front of the class.	3.71	1.006	58.1
I ask pupils to explain the solution to each other.	3.42	.720	42.0
I often use the cooperative group work for problem solving.	3.39	1.145	51.6

Table 4.

In case of unsuccessful problem solving

Item	Mean	Standard deviation	Affirmation is true (%)
I ask pupils to recall previous knowledge related with the problem.	3.32	1.077	41.9
I ask pupils to reread the problem.	4.45	.810	87.1
I ask pupils to tell what difficulties he/she has.	4.10	1.012	77.4
I give them hints which could help, but I don't tell the steps of the solution.	4.19	.910	74.2
I write the solution to the blackboard.	2.00	1.065	22.6
I tell him/her to try more methods.	3.45	.925	45.2

Table 5.

Selecting the problems given in the classroom

Item	Mean	Standard deviation	Affirmation is true (%)
I take in consideration the type of problems given at national tests.	2.84	1.068	25.8
I choose interesting problems.	4.03	.795	71.0
I give problems related with the everyday life.	4.10	.978	71.0
I choose problems which can be solved by almost all pupils from my classroom.	4.03	.912	67.7
I choose suitable problem for each pupil.	3.58	.992	48.4

Table 6.

Comparing teachers' problem solving behaviour and their expectations to pupils problem solving

Item	Teacher behaviour		Teacher's expectation to pupils		Correlation coefficient
	Mean	Standard deviation	Mean	Standard deviation	
Reformulating the text of the problem with ones own words.	3.13	1.088	3.97	1.016	.396*
Denoting the data and the relations between these data.	3.97	1.080	4.32	.945	.696***
Solving the problem with more methods.	2.48	.926	2.94	1.031	.452*
Checking if the solution is correct.	3.97	1.048	4.42	.765	.308
Checking if all the data are used.	4.16	.779	4.16	.969	.538**

* significance level .05, ** significance level .01, *** significance level .001

2.3. Discussion

When faced with a problem, self-regulated learners begin to analyze the task in order to identify the requirements of it (Pintrich, 2000; Schunk, 2000). Understanding the text of the problem is very important. Most of the teachers ask their pupils to read the text carefully (mean 4.61, standard deviation .803), three quarters of them request pupils to extract the known and unknown data respective the relations between these data (mean 4.32, standard deviation 1.016) and to reformulate the text of the problem (mean 3.97, standard deviation .945) – see Table 1. There is a strong correlation between teachers' behaviour and their expectation from pupils in case of denoting the data and the relations between these data (correlation coefficient .696) – Table 6. There is a mild correlation between teachers' behaviour

and their expectation from pupils in case of reformulating the text of the problem (correlation coefficient .396) – Table 6. In case of the teachers' behaviour the mean is less than in case of their expectation from pupils in case of both affirmations (Table 6).

Self-control and self-monitoring of the cognitive strategies, motivation, and behaviour are also important. While solving mathematics problems "control has to do with the decisions and actions undertaken in analyzing and exploring problem conditions, planning courses of action, selecting and organizing strategies, monitoring actions and progress, checking outcomes and results, evaluating plans and strategies, revising and abandoning unproductive plans and strategies, and reflecting upon all decisions made and actions taken during the course of working on a problem." (Lester et al., 1989, p. 4) Most of the teachers ask their pupils to check if the solution is correct (mean 4.42, standard deviation .765), three quarters of the teachers request pupils to verify if they have used all the data of the problem (mean 4.16, standard deviation .969) – Table 2. When solving problems by themselves there are teachers who usually don't check if the solution is correct (mean 3.97, standard deviation 1.048) – Table 6. There is a strong correlation between teachers' behaviour and their expectation from pupils in case of checking if all the data are used (Table 6). Searching for more methods for solving the same problem and evaluating these methods in order to chose the most efficient one is essential for developing problem solving skills. Most of the teachers usually don't try to search for more solving methods (mean 2.48, standard deviation .926) and only less than half of the teachers ask their pupils to solve a problem with more methods (mean 2.94, standard deviation 1.031) – Table 6. Almost half of the teachers ask pupils to select the most efficient solution in case they have solved a problem in more different ways (mean 3.68, standard deviation 1.013) – Table 2.

"When thinking is articulated regularly, patterns of thinking develop that are iterative. Thinking cannot be articulated unless students reflect on the problem and the strategies they use to solve it; articulation, in turn, increase reflection, which leads to understanding." (Fennema et al., 1999, p. 188) Writing down the detailed solution or explaining the solution to the class or to a colleague helps pupils to verbalize their thinking. The most expected behaviour is to write down the detailed solution (mean 3.81, standard deviation .873) and to explain the solution in front of the class (mean 3.71, standard deviation 1.006) – Table 3.

Group work helps students to acquire higher motivation and performance; and to work more independently (Rojas-Drummond et al., 1998; Stevens and Slavin, 1992). During collaborated learning students need to explain their reasoning and they get feedback from their colleagues. Through critically examining others thinking and reasoning, participating in discussions, students learn to monitor their own thinking and to build adequate reasoning (Artzt & Yaloz-Femia, 1999). Only less than half of the teachers ask their pupils to explain the solution to each other (mean 3.42, standard deviation .720) or to work in groups (mean 3.39, standard deviation 1.145) – Table 3.

A person with high problem solving skills is not lost in case of unsuccessful problem solving; he/she has methods for reaching a solution. Rereading the problem, recalling previous knowledge and worked examples, verbalizing his/her difficulties with the problem all help to overcome the unsuccessfulness of the first attempt. Most of the teachers ask their pupils to reread the problem (mean 4.45, standard deviation .810); three quarters of them guide pupils to verbalize their difficulties (mean 4.10, standard deviation 1.012) and give them hints without giving the solution (mean 4.19, standard deviation .910). Less than half of the teachers ask pupils to recall previous knowledge related with the problem (mean 3.32, standard deviation 1.077) or try more methods (mean 3.45, standard deviation .925) - Table 4. Almost one quarter of the teachers write the solution of the blackboard instead of guiding pupils to discover this solution (Table 4).

Almost three quarters of the teachers choose interesting problems (mean 4.03, standard deviation .795) or problems with connection to everyday life (mean 4.10, standard deviation .978) – Table 5. Only one quarter take in consideration the type of the problems given on national tests. Usually the problems from the Romanian national tests are mathematically formulated; don't have any relation with pupils' real life (Marchis, 2009a). None of the problems from these tests are challenging (Marchis, 2009b), most of them cover only the knowledge, understanding, and application cognitive levels: to solve them it is only required to apply formulas or algorithms. Three third of the respondents choose the problems in that way that most of the pupils could solve it (mean 4.03, standard deviation .912), which not always adequate, as talented pupils are not challenged in this way. Almost half of the teachers try differentiating the difficulty of the problem according to the problem solving level of each pupil (mean 3.58, standard deviation .992).

Conclusions, limitations, and future implications

Three quarters of the teachers guide pupils in order to understand the problem and encourage them for self-control during problem solving. Only one third of the respondents encourage their students to solve the problems with more methods. Only half of the teachers ask their pupils to present the solution (write down the detailed solution, explain the solution in front of the class or to each other in groups). Three quarters of the respondents have a positive attitude and guidance in case their pupils can't solve a problem. Almost three quarters of the primary school teachers give interesting, real life close problems in class. Teachers expect more from their pupils as regarding problem solving behaviour than they do when they solve problems by themselves.

For more relevant conclusion the sample size should be increased and class observations should be done.

Teacher training courses should focus on developing primary school teachers' problem solving skills and giving them the adequate pedagogical methods and tools to be able to raise pupils' motivation and develop their problem solving skills.

Acknowledgement

The results presented in this paper were developed in the frame of the PNII - IDEI, Exploratory research projects, ID_2418, supported by UEFISCSU.

REFERENCES

Antonietti, A. (1991). Why does mental visualization facilitate problem solving? In R. H. Logie & M. Denis (Eds.), *Mental images in human cognition.* Elsevier: Amsterdam.

Artzt, A. & Yaloz-Femia, S. (1999). Mathematical reasoning during small-group problem solving. In: Stiff, L. & Curio, F. (eds.), *Developing Mathematical Reasoning in Grades K-12: 1999 Yearbook* (pp. 115-126)*,* National Council of Teachers of Mathematics, Reston VA.

Ballew, H. & Cunningham, J. (1982). Diagnosing Strengths and Weaknesses of Six-Grade Students in Solving Word Problems. *Journal for Research in Mathematics Education,* 13(3), 202-210.

Bernardo, A. (1999). Overcoming Obstacles to Understanding and Solving Word Problems in Mathematics. *Educational Psychology*, 19(2), 149-163.

Brown, C.A., Carpenter, T.P., Kouba, V.L., Lindqist, M.M., Silver, E.A. & Swafford, J.O. (1988). Secondary school results of the fourth NAEP mathematics assessment: algebra, geometry, mathematical methods and attitudes*. Mathematics Teacher,* 81, 337-347.

Brown, S.I., Cooney, T.J. & Jones, D. (1990). Mathematics teacher education. In. W. R. Houston, M. Haberman & J. Sikula (Eds.), *Handbook of research on teacher education* (pp. 639-656), New York: Macmillan.

DeCorte, E. & Verschaffel, L. (1985). Beginning first graders' initial representations of arithmetic word problems. *The Journal of Mathematical Behavior*, *4*, 3-21.

Edwards, S.A., Maloy, R.W. & Verock-O'Loughlin, R. (2002). *Ways of writing with young kids: Teaching creativity and conventions unconventionally*. Boston: Allyn & Bacon.

Ernest, P. (1988). The Impact of Beliefs on the Teaching of mathematics. In P. Ernest (Ed.), *Mathematics Teaching: The State of the art,* (pp. 249-254). London: Falmer.

Fennema, E., Sowder, J. & Carpenter, T.P. (1999). Creating classrooms that promote understanding. In: Fennema, E. & Romberg, T.A. (eds.), *Mathematics classroom that promote understanding* (pp. 185-199). Mahwah, NJ: Erlbaum.

Gregg, J. (1995). The tensions and contradictions of the school Mathematics tradition. *Journal for Research in Mathematics Education*, 26 (5), 442-466.

Hegarty, M., Mayer, R.E. & Monk, C.A. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology,* 87(1), 18-32.

Hersh, R. (1986). Some proposals for revising the philosophy of mathematics. In T. Tymoczko (Ed.), New Directions in the philosophy of mathematics (pp. 9-28). Boston: Birkhauser.

Higgins, K.M. (1997). The effect of year-long instruction in mathematical problem solving on middle-school students' attitudes, beliefs, and abilities. *Journal of Experimental Education, 66*(1), 5-29.

Kintsch, W. & Greeno, J.G. (1985). Understanding and solving word arithmetic problems. *Psychological Review*, *92*, 109-129.

Leader, L.F. & Middleton, J.A. (2004). Promoting critical-thinking dispositions by using problem solving in middle school mathematics. *Research in Middle Level Education, 28*(1), 55-71.

Lester, F.K., Garofalo, J. & Kroll, D.L. (1989). *The role metacognition in mathematical problem solving: A study of two grade seven classes* (Final report, NSF project MDR 85-5046). Bloomington: Indiana University, mathematics Education Development Center.

Marchis, I. (2009a). Comparative analysis of the mathematics problems given at international tests and at the Romanian national tests, *Acta Didactica Napocensia,* 2(2) 141-148.

Marchis, I. (2009b). A comparison of the national tests on mathematics for 12-13 years old pupils in romania and singapore, *Studia UBB Psychol.-Paed.*, LIV(2), 177-184.

Marchis, I. (2011). Primary school teachers' self-regulated learning skills, *Acta Didactica Napocensia,* 4(4), 11-18.

Mayer, R.E. (1983). *Thinking Problem Solving, Cognition.* W.H.

Mayer, R.E. & Hegarty, M. (1996). The process of understanding mathematical problems. In R.J. Sternberg & T. Ben-Zeev (eds.). *The nature of mathematical thinking* (pp. 29-53). Mahwah NJ: Lawrence Elbaum.

McLeod, D.B. (1992). Research on affect in mathematics education: reconceptualization. In D.A. Grouws (ed.), *Handbook of Research on mathematics teaching and learning,* New York Macmillan, 575-596.

Pintrich, P.R. (2000). The role of goal orientation in self-regulated learning. In: M. Boecaerts, P.R. Pintrich & M. Zeidner (eds.), *Handbook of self-regulation*. San Diego: Academic Press, 451-502.

Pólya, G. (1945). *How to Solve It*. Princeton University Press.

Pólya, G. (1957). *How to Solve It*. Doubleday.

Reusser, K. (1989). *Textual and situational factors in solving mathematical word problems*. (Research Rep. No 7). Bern: Universität Bern.

Ridlon, C.L. (2004). The effect of a problem centered approach on low achieving six graders. *Focus on learning problems in mathematics,* 28(4), 6-29.

Schoenfeld, A.H. (1985). *Mathematical problem-solving.* New York: Academic Press.

Schoenfeld, A.H. (1989). Explorations of students' mathematical beliefs and behaviour. *Journal for research in mathematics Education*, 20(4), 338-335.

Schunk, D.H. (2000). *Learning theories: An educational perspective* (3rd edition). Upper Saddle River, NJ: Prentice-Hall.

Stapel, E. (2011). "Translating Word Problems: Keywords." Purplemath. Available from http://www.purplemath.com/modules/translat.htm. Accessed 13 April 2011

Swanson, H.L. (2004). Working memory and phonological processing as predictors of children's mathematical problem solving at different ages, *Memory & Cognition,* 32(4), 648-661.

Rojas-Drummond, S., Hernandez, G., Velez, M. & Villagran, G. (1998). Cooperative learning and the appropriation of procedural knowledge by primary school children. *Learning and Instruction*, 8, 37-62.

Stevens, R.J. & Slavin, R.E. (1992). *The Cooperative Elementary School: Effects on students' achievement, attitudes, and social relations.*Baltimore, MD: Center for Research on Effective Schooling for Disadvantaged Students.

Vilenius-Tuohimaa, P.M., Acenola, K. & Nurmi, J.-E. (2008). The association between mathematical word problems and reading comprehension, *Educational Psychology,* 28(4), 409-426.

Wiest, L.R. (1996). *The Role of Fantasy and Real-World Problem Contexts in Fourth - and Six-grade Students' Mathematical Problem Solving*, Doctoral Dissertation, Indiana University.